

Table 16.1

## THETA FUNCTIONS

$\epsilon \backslash \alpha$	$\vartheta_8(\epsilon \backslash \alpha)$							$\alpha / \epsilon_1$
	0°	5°	10°	15°	20°	25°		
0°	0.00000 0000	0.00000 0000	0.00000 0000	0.00000 0000	0.00000 0000	0.00000 0000		90°
5	0.08715 5743	0.08732 1966	0.08782 4152	0.08867 3070	0.08988 7414	0.09149 5034		85
10	0.17364 8178	0.17397 9362	0.17497 9967	0.17667 1584	0.17909 1708	0.18229 6223		80
15	0.25881 9045	0.25931 2677	0.26080 4191	0.26332 6099	0.26693 4892	0.27171 4833		75
20	0.34202 0143	0.34267 2476	0.34464 3695	0.34797 7361	0.35274 9211	0.35907 2325		70
25	0.42261 8262	0.42342 4343	0.42586 0446	0.42998 1306	0.43588 2163	0.44370 5382		65
30	0.50000 0000	0.50095 3708	0.50383 6358	0.50871 3952	0.51570 1435	0.52497 0857		60
35	0.57357 6436	0.57467 0526	0.57797 7994	0.58357 6134	0.59159 9683	0.60225 0597		55
40	0.64278 7610	0.64401 3768	0.64772 1085	0.65399 8067	0.66299 9145	0.67495 6130		50
45	0.70710 6781	0.70845 5688	0.71253 4820	0.71944 3681	0.72935 6053	0.74253 3161		45
50	0.76604 4443	0.76750 5843	0.77192 5893	0.77941 4712	0.79016 4790	0.80446 5863		40
55	0.81915 2044	0.82071 4821	0.82544 2256	0.83345 4505	0.84496 1783	0.86028 0899		35
60	0.86602 5404	0.86767 7668	0.87267 6562	0.88115 1505	0.89332 9083	0.90955 1166		30
65	0.90630 7787	0.90803 6964	0.91326 9273	0.92214 2410	0.93489 7610	0.95189 9199		25
70	0.93969 2621	0.94148 5546	0.94691 1395	0.95611 4956	0.96935 0025	0.98700 0216		20
75	0.96592 5826	0.96776 8848	0.97334 6839	0.98281 0311	0.99642 3213	1.01458 4761		15
80	0.98480 7753	0.98668 6836	0.99237 4367	1.00202 5068	1.01591 0350	1.03444 0908		10
85	0.99619 4698	0.99809 5528	1.00384 9133	1.01361 2807	1.02766 2527	1.04641 6011		5
90	1.00000 0000	1.00190 8098	1.00768 3786	1.01748 5224	1.03158 9925	1.05041 7974		0
$\epsilon \backslash \alpha$	30°	35°	40°	45°	50°	55°		$\alpha / \epsilon_1$
0°	0.00000 0000	0.00000 0000	0.00000 0000	0.00000 0000	0.00000 0000	0.00000 0000		90°
5	0.09353 4894	0.09606 0073	0.09914 2353	0.10287 9331	0.10740 5819	0.11291 2907		85
10	0.18636 3367	0.19139 9811	0.19754 9961	0.20501 0420	0.21405 3194	0.22506 4618		80
15	0.27778 4006	0.28530 3629	0.29449 2321	0.30564 8349	0.31918 5434	0.33569 3043		75
20	0.36710 5393	0.37706 5455	0.38924 7478	0.40405 4995	0.42204 9614	0.44403 4769		70
25	0.45365 1078	0.46599 3521	0.48110 6437	0.49950 2749	0.52189 9092	0.54932 5515		65
30	0.53676 4494	0.55141 5176	0.56937 7735	0.59127 8602	0.61799 6720	0.65080 1843		60
35	0.61581 3814	0.63268 1725	0.65339 2178	0.67868 8658	0.70961 8904	0.74770 4387		55
40	0.69019 6708	0.70917 3264	0.73250 7761	0.76106 3101	0.79606 0581	0.83928 2749		50
45	0.75934 4980	0.78030 3503	0.80611 4729	0.83776 1607	0.87664 1114	0.92480 2089		45
50	0.82272 9031	0.84552 4503	0.87364 0739	0.90817 9128	0.95071 1025	1.00355 1297		40
55	0.87986 2121	0.90433 1298	0.93455 6042	0.97175 1955	1.01765 9399	1.07485 2509		35
60	0.93030 4365	0.95626 6326	0.98837 8598	1.02796 3895	1.07692 1759	1.13807 1621		30
65	0.97366 6431	1.03092 3589	1.03467 8996	1.07635 2410	1.12798 8100	1.19262 9342		25
70	1.00961 2870	1.03795 2481	1.07308 5074	1.11651 4503	1.17041 0792	1.23801 2299		20
75	1.03786 5044	1.06706 1179	1.10328 6100	1.14811 2152	1.20381 2008	1.27378 3626		15
80	1.05820 3585	1.08801 9556	1.12503 6391	1.17087 7087	1.22789 0346	1.29959 2533		10
85	1.07047 0366	1.10066 1511	1.13815 8265	1.18461 4727	1.24242 6337	1.31518 2322		5
90	1.07456 9932	1.10488 6686	1.14254 4218	1.18920 7115	1.24728 6586	1.32039 6454		0
$\epsilon \backslash \alpha$	60°	65°	70°	75°	80°	85°		$\alpha / \epsilon_1$
0°	0.00000 0000	0.00000 0000	0.00000 0000	0.00000 0000	0.00000 0000	0.00000 0000		90°
5	0.11968 1778	0.12814 8474	0.13904 1489	0.15372 0475	0.17522 3596	0.21321 7690		85
10	0.23861 4577	0.25558 9564	0.27747 6571	0.30706 5715	0.35063 9262	0.42844 3440		80
15	0.35604 4091	0.38160 3032	0.41467 2740	0.45960 9511	0.52633 5260	0.64743 4941		75
20	0.47120 6153	0.50544 4270	0.54994 7578	0.61082 7702	0.70219 9693	0.87146 4767		70
25	0.58332 3727	0.62633 5361	0.68254 9331	0.76005 8920	0.87783 8622	1.10111 6239		65
30	0.69160 6043	0.74345 9784	0.81164 3704	0.90647 6281	1.05251 4778	1.33612 3616		60
35	0.79525 0355	0.85596 1570	0.93630 8263	1.04907 2506	1.22511 1680	1.57526 8297		55
40	0.89344 6594	0.96294 9380	1.05553 5305	1.18666 0037	1.39412 6403	1.81633 9939		50
45	0.98538 4972	1.06350 5669	1.16824 3466	1.31788 6740	1.55769 2334	2.05616 7815		45
50	1.07026 6403	1.15670 0687	1.27329 7730	1.44126 6644	1.71363 1283	2.29072 3417		40
55	1.14731 5349	1.24161 0747	1.36953 6895	1.55522 4175	1.85953 2258	2.51529 0558		35
60	1.21579 4546	1.31733 9855	1.45580 7011	1.65814 9352	1.99285 2358	2.72469 4161		30
65	1.27502 0900	1.38304 3549	1.53099 8883	1.74846 0610	2.11103 3523	2.91357 4159		25
70	1.32438 1718	1.43795 3601	1.59408 7380	1.82467 1332	2.21162 7685	3.07668 6743		20
75	1.36335 0417	1.48140 2159	1.64417 0149	1.88545 5864	2.29242 2061	3.20921 2227		15
80	1.39150 0813	1.51284 3876	1.68050 3336	1.92971 0721	2.35155 6149	3.30704 7313		10
85	1.40851 9209	1.53187 4716	1.70253 2036	1.95660 6998	2.38762 2438	3.36705 9918		5
90	1.41421 3562	1.53824 6269	1.70991 3565	1.96563 0511	2.39974 3837	3.38728 7004		0

$$\sqrt{\sec \alpha} \vartheta_8(\epsilon_1 \backslash \alpha)$$

$$\epsilon^\circ = \frac{\pi}{K} 90^\circ$$

$$\epsilon_1^\circ = 90^\circ - \epsilon^\circ$$

$$\alpha = \arcsin \sqrt{m}$$

$$\vartheta_8(u|m) = \vartheta_8(\epsilon^\circ \backslash \alpha^\circ)$$

In calculating elliptic functions from theta functions, when the modular angle exceeds about 60°, use the descending Landen transformation 16.12 to induce dependence on a smaller modular angle.

Compiled from E. P. Adams and R. L. Hipsley, Smithsonian mathematical formulae and tables of elliptic functions, 3d reprint (The Smithsonian Institution, Washington, D.C., 1957) (with permission).

## THETA FUNCTIONS

Table 16.1

		$\vartheta_n(e \backslash \alpha)$						
$e \backslash \alpha$		0°	5°	10°	15°	20°	25°	$\alpha/\epsilon_1$
0°	1		1.00000 00000	1.00000 00000	1.00000 00000	1.00000 00000	1.00000 00000	90
5	1		1.00001 44942	1.00005 83670	1.00013 28199	1.00023 99605	1.00038 29783	85
10	1		1.00005 75362	1.00023 16945	1.00052 72438	1.00095 25510	1.00152 02770	80
15	1		1.00012 78184	1.00051 47160	1.00117 12875	1.00211 61200	1.00357 73404	75
20	1		1.00022 32051	1.00089 88322	1.00204 53820	1.00369 53131	1.00589 77438	70
25	1		1.00034 07982	1.00137 23717	1.00312 29684	1.00564 21475	1.00900 49074	65
30	1		1.00047 70246	1.00192 09464	1.00437 13049	1.00789 74700	1.01260 44231	60
35	1		1.00062 77451	1.00252 78880	1.00575 24612	1.01039 27539	1.01658 69227	55
40	1		1.00078 83803	1.00317 47551	1.00722 44718	1.01305 21815	1.02083 14013	50
45	1		1.00095 40492	1.00384 18928	1.00874 26104	1.01579 49474	1.02520 88930	45
50	1		1.00111 97181	1.00450 90305	1.01026 07491	1.01853 77143	1.02958 63905	40
55	1		1.00128 03532	1.00515 58975	1.01173 27599	1.02119 71444	1.03383 08852	35
60	1		1.00143 10738	1.00576 28392	1.01311 39167	1.02369 24323	1.03781 34098	30
65	1		1.00156 73002	1.00631 14139	1.01436 22536	1.02594 77596	1.04141 29561	25
70	1		1.00168 48932	1.00678 49535	1.01543 98405	1.02789 45992	1.04452 01522	20
75	1		1.00178 02800	1.00716 90696	1.01631 39354	1.02947 37972	1.04704 05862	15
80	1		1.00185 05621	1.00745 20912	1.01695 79795	1.03063 73701	1.04889 76746	10
85	1		1.00189 36042	1.00762 54187	1.01735 24037	1.03134 99632	1.05003 49895	5
90	1		1.00190 80984	1.00768 37857	1.01748 52237	1.03158 99246	1.05041 79735	0
$e \backslash \alpha$		30°	35°	40°	45°	50°	55°	$\alpha/\epsilon_1$
0°	1.00000 00000		1.00000 00000	1.00000 00000	1.00000 00000	1.00000 00000	1.00000 00000	90°
5	1.00056 64294		1.00079 66833	1.00108 26253	1.00143 67802	1.00187 71775	1.00243 05914	85
10	1.00224 85079		1.00316 25308	1.00429 76203	1.00570 30665	1.00745 17850	1.00964 88003	80
15	1.00499 51300		1.00702 56701	1.00954 73402	1.01267 06562	1.01655 47635	1.02143 61311	75
20	1.00872 28461		1.01226 87413	1.01667 23379	1.02212 67193	1.02891 00179	1.03743 56974	70
25	1.01331 83978		1.01873 24599	1.02545 62012	1.03378 46028	1.04414 27466	1.05716 29130	65
30	1.01864 21583		1.02622 04548	1.03563 21191	1.04729 03271	1.06179 07561	1.08002 00285	60
35	1.02453 23743		1.03450 52308	1.04689 09786	1.06223 37524	1.08131 84270	1.10531 40947	55
40	1.03081 00797		1.04333 50787	1.05889 07481	1.07816 10137	1.10213 29153	1.13227 78297	50
45	1.03728 45330		1.05244 17208	1.07126 68617	1.09458 82886	1.12360 21058	1.16009 27802	45
50	1.04375 90125		1.06154 84606	1.08364 32917	1.11101 64844	1.14507 37802	1.18791 40899	40
55	1.05003 67930		1.07037 85902	1.09564 39724	1.12694 63970	1.16589 54205	1.21489 61356	35
60	1.05592 71242		1.07866 37978	1.10690 42279	1.14189 38846	1.18543 40490	1.24021 82552	30
65	1.06125 10260		1.08615 23221	1.11708 18582	1.15540 45920	1.20309 54999	1.26310 97835	25
70	1.06584 67280		1.09261 66042	1.12586 75438	1.16706 77783	1.21834 25328	1.28287 36204	20
75	1.06957 45853		1.09786 02047	1.13299 42539	1.17652 88244	1.23071 12287	1.29890 75994	15
80	1.07232 13226		1.10172 37756	1.13824 53698	1.18350 00363	1.23982 51648	1.31072 29838	10
85	1.07400 34764		1.10408 99048	1.14146 12760	1.18776 94140	1.24540 69243	1.31795 95033	5
90	1.07456 99318		1.10488 66859	1.14254 42177	1.18920 71150	1.24728 65857	1.32039 64540	0
$e \backslash \alpha$		60°	65°	70°	75°	80°	85°	$\alpha/\epsilon_1$
0°	1.00000 00000		1.00000 00000	1.00000 00000	1.00000 00000	1.00000 00000	1.00000 00000	90°
5	1.00313 85295		1.00406 92257	1.00534 44028	1.00720 88997	1.01026 06485	1.01663 88247	85
10	1.01245 94672		1.01615 50083	1.02121 95717	1.02862 79374	1.04076 43440	1.06618 38299	80
15	1.02768 16504		1.03589 51569	1.04715 56657	1.06363 90673	1.09068 07598	1.14751 59063	75
20	1.04834 57003		1.06269 75825	1.08238 38086	1.11122 86903	1.15864 11101	1.25875 62174	70
25	1.07382 76019		1.09575 73598	1.12585 71388	1.17001 24008	1.24276 19421	1.39725 25218	65
30	1.10335 71989		1.13408 00433	1.17627 97795	1.23826 96285	1.34068 05139	1.55957 26706	60
35	1.13604 11010		1.17651 31946	1.23214 31946	1.31398 80140	1.44960 33094	1.74151 57980	55
40	1.17088 93642		1.22176 77148	1.29176 91861	1.39491 71251	1.56636 90138	1.93815 19599	50
45	1.20684 51910		1.26848 10938	1.35335 85717	1.47863 07744	1.68752 66770	2.14389 95792	45
50	1.24281 67937		1.31523 31927	1.41504 43413	1.56259 67789	1.80942 88493	2.35264 71220	40
55	1.27771 04815		1.36060 17261	1.47494 78592	1.64425 25175	1.92833 82823	2.55792 12198	35
60	1.31046 39783		1.40320 31647	1.53123 64694	1.72108 41609	2.04054 54606	2.75309 84351	30
65	1.34007 89457		1.44173 53793	1.58218 06891	1.79070 70015	2.14249 29245	2.93165 25995	25
70	1.36565 16965		1.47501 81348	1.62620 90720	1.85094 39670	2.23090 12139	3.08742 47870	20
75	1.38640 11169		1.50203 00916	1.66195 87940	1.89989 92030	2.30289 04563	3.21489 91220	15
80	1.40169 28947		1.52194 10514	1.68832 00831	1.93602 35909	2.35609 12550	3.30946 52989	10
85	1.41105 92570		1.53413 83232	1.70447 27784	1.95816 92561	2.38873 86793	3.36764 82512	5
90	1.41421 35624		1.53824 62687	1.70991 35651	1.96563 05108	2.39974 38370	3.38728 70037	0

$$e = \frac{u}{K} 90^\circ \quad \epsilon_1 = 90^\circ - e \quad \sqrt{\sec \alpha} \vartheta_n(e_1 \backslash \alpha) \quad \alpha = \arcsin \frac{\sqrt{m}}{\sqrt{m}} \quad \vartheta_n(u|m) = \vartheta_n(e \backslash \alpha)$$

In calculating elliptic functions from theta functions, when the modular angle exceeds about  $60^\circ$ , use the descending Landen transformation 16.12 to induce dependence on a smaller modular angle.